MATHEMATICS - CET 2021 - VERSION Code - B2
SOLUTION

1. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$ then $(AB)'$ is equal to

(A) $\begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$ (B) $\begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$ (C) $\begin{bmatrix} -3 & 7 \\ 10 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} -3 & 7 \\ 10 & -2 \end{bmatrix}$

Ans (B)

$$AB = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-6+1 & 1-4+1 \\ 4+3+3 & 2+2+3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$$

Then $(AB)' = \begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$

2. Let M be 2×2 symmetric matrix with integer entries, then M is invertible if

(A) the first column of M is the transpose of second row of M

(B) the second row of M is transpose of first column of M

(C) M is a diagonal matrix with non-zero entries in the principal diagonal

(D) The product of entries in the principal diagonal of M is the product of entries in the other diagonal

Ans (C)

For matrix to be invertible, determinant must not be equal to zero, that is matrix should be non-singular.

Let $M = \begin{pmatrix} a & h \\ h & b \end{pmatrix}$

$|M| = ab - h^2 \neq 0$ i.e., $ab \neq h^2$

Therefore, 'M' is a diagonal matrix with non-zero entries in the main diagonal of M is not the square of an integer.

3. If A and B are matrices of order 3 and $|A| = 5$, $|B| = 3$ then $|3AB|$ is

(A) 425 (B) 405 (C) 565 (D) 585

Ans (B)

Give $|A| = 5 |B| = 3$

Then $|3AB| = 3^3 |A||B|$ $\quad (\because |KA| = K^n |A|)$

$= 3^3 (5)(3)$

$= 27(15) = 405$

4. If A and B are invertible matrices then which of the following is not correct?

(A) adj $A = |A| A^{-1}$ (B) $\det(A^{-1}) = [\det(A)]^{-1}$

(C) $(AB)^{-1} = B^{-1}A^{-1}$ (D) $(A + B)^{-1} = B^{-1} + A^{-1}$

Ans (D)

Since $(A+B)' = A' + B'$ but $(A+B)^{-1} = B^{-1} + A^{-1}$ is not correct.

5. If $f(x) = \begin{vmatrix} \cos x & 1 & 0 \\ 0 & 2\cos x & 3 \\ 0 & 1 & 2\cos x \end{vmatrix}$ then $\lim\limits_{x\to\pi} f(x) =$

(A) – 1 (B) 1 (C) 0 (D) 3

Ans (A)

Given $f(x) = \begin{vmatrix} \cos x & 1 & 0 \\ 0 & 2\cos x & 3 \\ 0 & 1 & 2\cos x \end{vmatrix}$

Expand along first column

$= \cos x\,(4\cos^2 x - 3) - 0\,(\;) + 0\,(\;)$

$= 4\cos^3 x - 3\cos x$

$f(x) = \cos 3x$

$\therefore \lim\limits_{x\to\pi} f(x) = \cos 3\pi$

$= -1$

6. If $x^3 - 2x^2 - 9x + 18 = 0$ and $A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & x & 6 \\ 7 & 8 & 9 \end{vmatrix}$ then the maximum value of A is

(A) 96 (B) 36 (C) 24 (D) 120

Ans (A)

$x^3 - 2x^2 - 9x + 18 = 0$

$\Rightarrow x = 2, 3, -3$

$x = 2,\; A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 2 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 36$

$x = 3,\; A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 3 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 24$

$x = -3,\; A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & -3 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 96$

7. At x = 1, the function $f(x) = \begin{cases} x^3 - 1 & 1 < x < \infty \\ x - 1 & -\infty < x \le 1 \end{cases}$ is

(A) continuous and differentiable (B) continuous and non-differentiable

(C) discontinuous and differentiable (D) discontinuous and non-differentiable

Ans (B)

At x = 1, f(1) = LHL = RHL = 0, $\therefore$ f(x) is continuous

$\text{RHD} = R\{f'(x)\} = 3x^2$

$= R\{f'(1)\} = 3(1)^2 = 3$

LHD = L{f ′(x)} = 1

= L{f ′(1)} = 1

LHD ≠ RHD

Non-differentiable

8. If $y=(\cos x^2)^2$, then $\frac{dy}{dx}$ is equal to

(A) $-4x\sin 2x^2$ (B) $-x\sin x^2$ (C) $-2x\sin 2x^2$ (D) $-x\cos 2x^2$

Ans (C)

$y=(\cos x^2)^2$

$\Rightarrow \frac{dy}{dx}=2\cos(x^2)(-\sin(x^2))2x=-2x\sin(2x^2)$

9. For constant a, $\frac{d}{dx}(x^x+x^a+a^x+a^a)$ is

(A) $x^x(1+\log x)+ax^{a-1}$

(B) $x^x(1+\log x)+ax^{a-1}+a^x\log a$

(C) $x^x(1+\log x)+a^a(1+\log x)$

(D) $x^x(1+\log x)+a^a(1+\log a)+ax^{a-1}$

Ans (B)

$\frac{d}{dx}(x^x+x^a+a^x+a^a)$

$=x^x(1+\log x)+ax^{a-1}+a^x\log a$

10. Consider the following statements:

Statement 1: If $y=\log_{10}x+\log_e x$ then $\frac{dy}{dx}=\frac{\log_{10}e}{x}+\frac{1}{x}$

Statement 2: $\frac{d}{dx}(\log_{10}x)=\frac{\log x}{\log 10}$ and $\frac{d}{dx}(\log_e x)=\frac{\log x}{\log e}$

(A) Statement 1 is true, statement 2 is false

(B) Statement 1 is false, statement 2 is true

(C) Both statements 1 and 2 are true

(D) Both statements 1 and 2 are false

Ans (A)

$y=\frac{\log_e x}{\log_e 10}+\log_e x=\log_{10}e\log_e x+\log_e x$

$\frac{dy}{dx}=\frac{\log_{10}e}{x}+\frac{1}{x}$

$\frac{d}{dx}(\log_{10}x)\neq\frac{\log x}{\log 10}$; $\frac{d}{dx}(\log_e x)\neq\frac{\log_e x}{\log e}$

11. If the parametric equation of a curve is given by $x=\cos\theta+\log\tan\frac{\theta}{2}$ and y = sin θ, then the points for which $\frac{dy}{dx}=0$ are given by

(A) $\theta=\frac{n\pi}{2}, n\in z$

(B) $\theta=(2n+1)\frac{\pi}{2}, n\in z$

(C) $\theta=(2n+1)\pi, n\in z$

(D) $\theta=n\pi, n\in z$

Ans (D)

$x=\cos\theta+\log\tan\left(\frac{\theta}{2}\right)$

$$\frac{dx}{d\theta} = -\sin\theta + \frac{1}{\tan\frac{\theta}{2}}\sec^2\frac{\theta}{2}.\frac{1}{2}$$

$$= -\sin\theta + \frac{1}{\sin\theta}$$

$$\frac{dx}{d\theta} = \frac{\cos^2\theta}{\sin\theta}$$

$y = \sin\theta$

$$\frac{dy}{d\theta} = \cos\theta$$

$$\frac{dy}{dx} = \frac{\cos\theta}{\frac{\cos^2\theta}{\sin\theta}} = \tan\theta \quad \Rightarrow \tan\theta = 0$$

$\theta = n\pi, n \in z$

12. If $y = (x-1)^2 (x-2)^3 (x-3)^5$ then $\frac{dy}{dx}$ at $x = 4$ is equal to

(A) 108 (B) 54 (C) 36 (D) 516

Ans (D)

$y = (x-1)^2 (x-2)^3 (x-3)^5$

$\log y = 2\log(x-1) + 3\log(x-2) + 5\log(x-3)$

$$\frac{1}{y}.\frac{dy}{dx} = \frac{2}{x-1} + \frac{3}{x-2} + \frac{5}{x-3}$$

$$\frac{dy}{dx} = y\left[\frac{2}{x-1} + \frac{3}{x-2} + \frac{5}{x-3}\right]$$

$$\left(\frac{dy}{dx}\right)_{x=4} = 3^2.2^3.1^5\left[\frac{2}{3} + \frac{3}{2} + \frac{5}{1}\right]$$

$$= 9\times 8\left[\frac{4+9+30}{6}\right]$$

$= 12 \times 43$

$= 516$

13. A particle starts from rest and its angular displacement (in radians) is given by $\theta = \frac{t^2}{20} + \frac{t}{5}$. If the angular velocity at the end of $t = 4$ is k, then the value of 5 k is

(A) 0.6 (B) 5 (C) 5 k (D) 3

Ans (D)

$$\theta = \frac{t^2}{20} + \frac{t}{5}$$

$$\frac{d\theta}{dt} = \frac{t}{10} + \frac{1}{5}$$

Given, $v_{t=4} = k \Rightarrow \frac{2}{5} + \frac{1}{5} = k$

$$\Rightarrow k = \frac{3}{5} \Rightarrow 5k = 3$$

14. If the parabola $y = \alpha x^2 - 6x + \beta$ passes through the point (0, 2) and has its tangent at $x = \frac{3}{2}$ parallel to x axis, then

(A) $\alpha = 2, \beta = -2$ (B) $\alpha = -2, \beta = 2$ (C) $\alpha = 2, \beta = 2$ (D) $\alpha = -2, \beta = -2$

Ans (C)

$y = \alpha x^2 - 6x + \beta$ …(1)

$\frac{dy}{dx} = 2\alpha x - 6$

$\frac{dy}{dx} = 2\alpha x - 6 = 0$ [$\because$ parallel to x-axis]

$\left(\frac{dy}{dx}\right)_{x=\frac{3}{2}} = 3\alpha - 6 = 0 \quad \Rightarrow \alpha = 2$

(0, 2) lies on (1)

$(1) \Rightarrow 2 = 0 - 0 - \beta$

$\therefore \beta = 2$

15. The function $f(x) = x^2 - 2x$ is strictly decreasing in the interval

(A) $(-\infty, 1)$ (B) $(1, \infty)$ (C) R (D) $(-\infty, \infty)$

Ans (A)

$f(x) = x^2 - 2x$

$f'(x) = 2x - 2$

Given $f'(x) < 0 \quad \Rightarrow 2x - 2 < 0$

$x < 1$

$\therefore x \in (-\infty, 1)$

16. The maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$ is

(A) 1 (B) 23 (C) 5 (D) – 23

Ans (C)

Given $y = -x^3 + 3x^2 + 2x - 27$

$m = \frac{dy}{dx} = -3x^2 + 6x + 2$

m is max $\Rightarrow \frac{dm}{dx} = 0$

$\Rightarrow -6x + 6 = 0 \Rightarrow x = 1$

$\therefore$ Max slope is $= -3 + 6 + 2$

$= 5$

Verification : $\frac{d^2m}{dx^2} = -6 < 0$ at $x = 1$

17. $\int \frac{x^3 \sin(\tan^{-1}(x^4))}{1 + x^8} dx$ is equal to

(A) $\frac{-\cos(\tan^{-1}(x^4))}{4} + C$ (B) $\frac{\cos(\tan^{-1}(x^4))}{4} + C$

(C) $\frac{-\cos(\tan^{-1}(x^3))}{3} + C$ (D) $\frac{\sin(\tan^{-1}(x^4))}{4} + C$

Ans (A)

$$\int \frac{x^3 \sin(\tan^{-1}(x^4))}{1+x^8}dx$$
$$= \frac{1}{4}\int \sin(t)dt$$
$$= -\frac{1}{4}\cos t + c$$
$$= -\frac{1}{4}\cos(\tan^{-1} x^4) + c$$

Put $t = \tan^{-1} x^4$

$dt = \frac{1}{1+x^8} \cdot 4x^3 dx$

$\frac{1}{4}dt = \frac{x^3}{1+x^8}dx$

18. The value of $\int \frac{x^2 dx}{\sqrt{x^6 + a^6}}$ is equal to

(A) $\log\left|x^3 + \sqrt{x^6 + a^6}\right| + c$

(B) $\log\left|x^3 - \sqrt{x^6 + a^6}\right| + c$

(C) $\frac{1}{3}\log\left|x^3 + \sqrt{x^6 + a^6}\right| + c$

(D) $\frac{1}{3}\log\left|x^3 - \sqrt{x^6 + a^6}\right| + c$

Ans (C)

$$\int \frac{x^2}{\sqrt{x^6 + a^6}}dx = \int \frac{x^2}{\sqrt{(x^3)^2 + (a^3)^2}}dx$$
$$= \frac{1}{3}\int \frac{1}{\sqrt{t^2 + (a^3)^2}}dt$$
$$= \frac{1}{3}\log\left(t + \sqrt{t^2 + a^6}\right) + c$$
$$= \frac{1}{3}\log\left(x^3 + \sqrt{x^6 + a^6}\right) + c$$

Put $t = x^3$

$\frac{dt}{dx} = 3x^2$

$\frac{1}{3}dt = x^2 dx$

19. The value of $\int \frac{xe^x dx}{(1+x)^2}$ is equal to

(A) $e^x(1 + x) + c$ (B) $e^x(1 + x^2) + c$ (C) $e^x(1 + x)^2 + c$ (D) $\frac{e^x}{1+x} + c$

Ans (D)

$$\int e^x \frac{x}{(1+x)^2}dx = \int e^x \frac{(1+x-1)}{(1+x)^2}dx$$
$$= \int e^x \left(\frac{1}{(1+x)} - \frac{1}{(1+x)^2}\right)dx$$
$$= e^x \frac{1}{1+x} + c$$

$\int e^x (f(x) + f'(x))dx = e^x f(x) + c$

20. The value of $\int e^x \left[\frac{1+\sin x}{1+\cos x}\right]dx$ is equal to

(A) $e^x \tan\frac{x}{2} + c$ (B) $e^x \tan x + c$ (C) $e^x(1 + \cos x) + c$ (D) $e^x(1 + \sin x) + c$

Ans (A)

$$\int e^x \left(\frac{1+\sin x}{1+\cos x}\right)dx = \int e^x \left(\frac{1+\sin x}{2\cos^2(x/2)}\right)dx$$
$$= \int e^x \left(\frac{1}{2\cos^2(x/2)} + \frac{2\sin(x/2)\cos(x/2)}{2\cos^2(x/2)}\right)dx$$

$\int e^x (f(x) + f'(x))dx = e^x f(x) + c$

$$= \int e^x \left(\tan \frac{x}{2} + \frac{1}{2} \sec^2 \frac{x}{2} \right) dx$$

$$= e^x \tan \frac{x}{2} + c$$

21. If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$ where n is positive integer then $I_{10} + I_8$ is equal to

(A) 9 (B) $\frac{1}{7}$ (C) $\frac{1}{8}$ (D) $\frac{1}{9}$

Ans (D)

$$I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$$

$$I_{10} + I_8 = \int_0^{\frac{\pi}{4}} (\tan^{10} x + \tan^8 x) dx$$

$$= \int_0^{\frac{\pi}{4}} \tan^8 x (\tan^2 x + 1) \, dx$$

$$= \int_0^{\frac{\pi}{4}} \tan^8 x \sec^2 x dx$$

$$= \left. \frac{\tan^9 x}{9} \right)_0^{\frac{\pi}{4}}$$

$$= \frac{1}{9} \{1 - 0\} = \frac{1}{9}$$

Or

$$I_{10} + I_8 = \frac{1}{10 - 1} = \frac{1}{9}$$

22. The value of $\int_0^{4042} \frac{\sqrt{x} \, dx}{\sqrt{x} + \sqrt{4042 - x}}$ is equal to

(A) 4042 (B) 2021 (C) 8084 (D) 1010

Ans (B)

$$I = \int_0^{4042} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{4042 - x}} dx \qquad ...(1)$$

$$I = \int_0^{4042} \frac{\sqrt{4042 - x}}{\sqrt{4042 - x} + \sqrt{x}} \qquad ...(2)$$

Add (1) and (2)

$$\Rightarrow 2I = \int_0^{4042} \frac{\sqrt{x} + \sqrt{4042 - x}}{\sqrt{4042 - x} + \sqrt{x}} dx$$

$$= \int_0^{4042} 1 \, dx$$

$$= \left. x \right)_0^{4042}$$

$2I = 4042$

$I = 2021$

Or

$$I = \int_0^{4042} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{4042 - x}} dx = \frac{b-a}{2}$$

$$= \frac{4042 - 0}{2} = 2021$$

23. The area of the region bounded by $y = \sqrt{16 - x^2}$ and x-axis is

(A) 8 square units (B) 20π square units (C) 16π square units (D) 256π square units

Ans GRACE

$y = \sqrt{16 - x^2}$

$y^2 = 16 - x^2$

$x^2 + y^2 = 16$

$x^2 + y^2 = a^2$

$a^2 = 16$

Area of circle = πr^2

Area of circle and the x-axis

$= \frac{1}{2}\pi r^2 = \frac{1}{12}\pi(16) \quad = 8\pi$

24. If the area of the Ellipse is $\frac{x^2}{25} + \frac{y^2}{\lambda^2} = 1$ is 20π square units, then λ is

(A) ± 4 (B) ± 3 (C) ± 2 (D) ± 1

Ans (A)

$\frac{x^2}{25} + \frac{y^2}{\lambda^2} = 1$

Area of Ellipse is π ab = 20 π

a = 5, b = $|\lambda|$

$\pi(5)(|\lambda|) = 20\pi$

$|\lambda| = 4 \Rightarrow \lambda = \pm 4$

25. Solution of Differential Equation xdy − ydx = 0 represents

(A) A rectangular Hyperbola (B) Parabola whose vertex is at origin

(C) Straight line passing through origin (D) A circle whose centre is origin

Ans (C)

$xdy - ydx = 0$

$xdy = ydx$

$\frac{1}{y}dy = \frac{1}{x}dx$

$\log y = \log x + \log c$

$\log y = \log(xc)$

y = xc is straight line passing through origin.

26. The number of solutions of $\frac{dy}{dx}=\frac{y+1}{x-1}$ when y(1) = 2 is

(A) three (B) one (C) infinite (D) two

Ans (B)

$\frac{dy}{dx}=\frac{y+1}{x-1}$

$\frac{1}{y+1}dy=\frac{1}{x-1}dx$

$\log(y+1)=\log(x-1)-\log c$

$\log(y+1)+\log c=\log(x-1)$

$(y+1)c=x-1$

$c=\frac{x-1}{y+1}$

$y(1)=2$

x = 1, y = 2

$c=\frac{1-1}{2+1}$

c = 0

So, the required solution is x − 1 = 0 hence, only one solution.

27. A vector $\vec{a}$ makes equal acute angles on the coordinate axis. Then the projection of vector $\vec{b}=5\hat{i}+7\hat{j}-\hat{k}$ on $\vec{a}$ is

(A) $\frac{11}{15}$ (B) $\frac{11}{\sqrt{3}}$ (C) $\frac{4}{5}$ (D) $\frac{3}{5\sqrt{3}}$

Ans (B)

$\vec{b}=5\hat{i}+7\hat{j}-\hat{k}$ $\vec{a}=\hat{i}+\hat{j}+\hat{k}$

Projection of a $\vec{b}$ on $\vec{a}=\frac{\vec{b}\cdot\vec{a}}{|a|}$

$=\frac{5+7-1}{\sqrt{3}}$

$=\frac{11}{\sqrt{3}}$

28. The diagonals of a parallelogram are the vectors $3\hat{i}+6\hat{j}-2\hat{k}$ and $-\hat{i}-2\hat{j}-8\hat{k}$ then the length of the shorter side of parallelogram is

(A) $2\sqrt{3}$ (B) $\sqrt{14}$ (C) $3\sqrt{5}$ (D) $4\sqrt{3}$

Ans Grace

$\vec{a}+\vec{b}=3\hat{i}+6\hat{j}-2\hat{k}$

$\vec{a}+\vec{b}+\vec{a}-\vec{b}=2\hat{i}+4\hat{j}-10\hat{k}$

$2\vec{a}=2\hat{i}+4\hat{j}-10\hat{k}$

$\vec{a}=\hat{i}+2\hat{j}-5\hat{k}$

$|\vec{a}|=\sqrt{1+4+25}$

$|\vec{a}|=\sqrt{30}$

$\vec{a}-\vec{b}=-\hat{i}-2\hat{j}-8\hat{k}$

$\vec{a}+\vec{b}-\vec{a}+\vec{b}=4\hat{i}+8\hat{j}+6\hat{k}$

$2\vec{b}=4\hat{i}+8\hat{j}+6\hat{k}$

$\vec{b}=2\hat{i}+4\hat{j}+3\hat{k}$

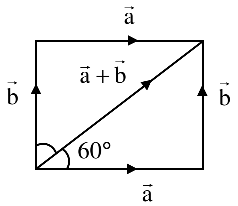
$|\vec{b}|=\sqrt{4+16+9}$

$|\vec{b}|=\sqrt{29}$

29. If $\vec{a}\cdot\vec{b}=0$ and $\vec{a}+\vec{b}$ makes an angle 60° with $\vec{a}$, then

(A) $|\vec{a}|=2|\vec{b}|$ (B) $2|\vec{a}|=|\vec{b}|$ (C) $|\vec{a}|=\sqrt{3}|\vec{b}|$ (D) $\sqrt{3}|\vec{a}|=|\vec{b}|$

Ans (D)



$$\tan 60=\frac{|\vec{b}|}{|\vec{a}|}$$

$$\Rightarrow \sqrt{3}=\frac{|\vec{b}|}{|\vec{a}|}$$

$$\sqrt{3}|\vec{a}|=|\vec{b}|$$

30. If the area of the parallelogram with $\vec{a}$ and $\vec{b}$ as two adjacent sides is 15 sq. units then the area of the parallelogram having $3\vec{a}+2\vec{b}$ and $\vec{a}+3\vec{b}$ as two adjacent side in sq. units is

(A) 45 (B) 75 (C) 105 (D) 120

Ans (C)

Given $|\vec{a}\times\vec{b}|=15$

If the sides are $(3\vec{a}+2\vec{b})$ and $(\vec{a}+3\vec{b})$

$$\text{Area}=\left|(3\vec{a}+2\vec{b})\times(\vec{a}+3\vec{b})\right|$$

$$=\left|2(\vec{b}\times\vec{a})+9(\vec{a}\times\vec{b})\right|$$

$$=\left|-2(\vec{a}\times\vec{b})+9(\vec{a}\times\vec{b})\right|$$

$$=\left|7(\vec{a}\times\vec{b})\right|$$

$$=7\left|\vec{a}\times\vec{b}\right|$$

= 7(15)

= 105 sq. units

31. The equation of the line joining the points (–3, 4, 11) and (1, –2, 7) is

(A) $\frac{x+3}{2}=\frac{y-4}{3}=\frac{z-11}{4}$ (B) $\frac{x+3}{-2}=\frac{y-4}{3}=\frac{z-11}{2}$

(C) $\frac{x+3}{-2}=\frac{y+4}{3}=\frac{z+11}{4}$ (D) $\frac{x+3}{2}=\frac{y+4}{-3}=\frac{z+11}{2}$

Ans (B)

We know that the equation of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$

⇒ here given points are (–3, 4, 11) and (1, –2, 7)

Now the equation of the line is $\frac{x+3}{1+3}=\frac{y-4}{-2-4}=\frac{z-11}{7-11}$

i.e., $\frac{x+3}{4}=\frac{y-4}{-6}=\frac{z-11}{-4}$

Clearly the dr's of the line are proportional to –2, 3, 2

∴ the required equation of line is $\frac{x+3}{-2}=\frac{y-4}{3}=\frac{z-11}{2}$

32. The angle between the lines whose direction cosines are $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{-\sqrt{3}}{2}\right)$ is

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

Ans (C)

Given $(l_1, m_1, n_1) = \left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right)$ and $(l_2, m_2, n_2) = \left(\frac{\sqrt{3}}{4}, \frac{1}{4}, -\frac{\sqrt{3}}{2}\right)$

We have $\cos\theta = \frac{|l_1 l_2 + m_1 m_2 + n_1 n_2|}{\sqrt{l_1^2 + m_1^2 + n_1^2} \times \sqrt{l_2^2 + m_2^2 + n_2^2}}$

$\Rightarrow \cos\theta = \frac{\left|\frac{3}{16} + \frac{1}{16} - \frac{3}{4}\right|}{\sqrt{1}\sqrt{1}}$

$\Rightarrow \cos\theta = \left|-\frac{1}{2}\right| \quad \Rightarrow \cos\theta = \frac{1}{2} \quad \Rightarrow \theta = \frac{\pi}{3}$

33. If a plane meets the coordinate axes at A, B and C in such a way that the centroid of triangle ABC is at the point (1, 2, 3), then the equation of the plane is

(A) $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 1$ (B) $\frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 1$ (C) $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = \frac{1}{3}$ (D) $\frac{x}{1} - \frac{y}{2} + \frac{z}{3} = -1$

Ans (B)

Let the plane meets the coordinate axes at the points A(a, 0, 0), B(0, b, 0) and C(0, 0, c) respectively

The centroid of the triangle $ABC = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

Given centroid = (1, 2, 3)

$\Rightarrow \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (1, 2, 3)$

$\Rightarrow$ a = 3, b = 6, c = 9

Now the equation of the plane in intercept from is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

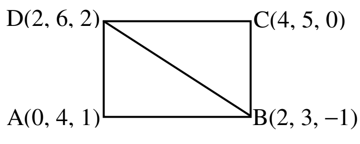
$\Rightarrow \frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 1$

34. The area of the quadrilateral ABCD, when A(0, 4, 1) B(2, 3, −1) C(4, 5, 0) and D(2, 6, 2) is equal to

(A) 9 sq. units (B) 18 sq. units (C) 27 sq. units (D) 81 sq. units

Ans (A)

Required area of quadrilateral is



= (Area of Δ ABD) + (Area of ΔBCD)

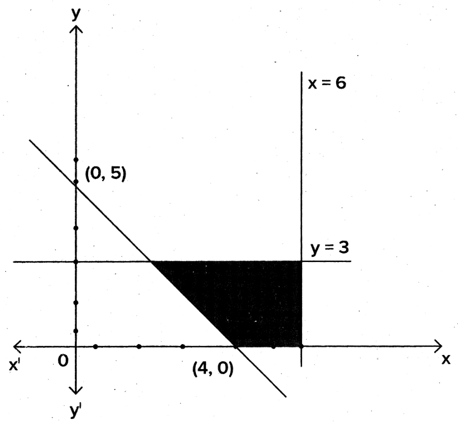
$= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AD}| + \frac{1}{2}|\overrightarrow{CB} \times \overrightarrow{CD}|$

$= \frac{1}{2}|3\hat{i} - 6\hat{j} + 6\hat{k}| + \frac{1}{2}|-3\hat{i} + 6\hat{j} - 6\hat{k}|$

$= \frac{1}{2}\sqrt{81} + \frac{1}{2}\sqrt{81}$

$= \frac{1}{2}(9) + \frac{1}{2}(9) \quad = \frac{1}{2}(18) \quad = 9$ sq units.

35. The shaded region is the solution set of the inequalities



(A) $5x + 4y \ge 20, x \le 6, y \ge 3, x \ge 0, y \ge 0$
(B) $5x + 4y \le 20, x \le 6, y \le 3, x \ge 0, y \ge 0$
(C) $5x + 4y \ge 20, x \le 6, y \le 3, x \ge 0, y \ge 0$
(D) $5x + 4y \ge 20, x \ge 6, y \le 3, x \ge 0, y \ge 0$

Ans (C)

Clearly the point (4, 1) exists in the common solution region.

$\Rightarrow$ (4, 1) must satisfy all the inequalities

Clearly (4, 1) satisfies all the inequalities of option (C)

36. Given that A and B are two events such that $P(B) = \frac{3}{5}$ $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$ then P(A) =

(A) $\frac{3}{10}$
(B) $\frac{1}{2}$
(C) $\frac{1}{5}$
(D) $\frac{3}{5}$

Ans (B)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$\frac{4}{5} = P(A) + \frac{3}{5} - \left(\frac{1}{2}\right)\left(\frac{3}{5}\right) \Rightarrow P(A) = \frac{1}{2}$

37. If A, B and C are three independent events such that P(A) = P(B) = P(C) = P then P(at least two of A, B, C occur) =

(A) $P^3 - 3P$
(B) $3P - 2P^2$
(C) $3P^2 - 2P^3$
(D) $3P^2$

Ans (C)

P(at least 2 of A,B,C occur) = $P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A' \cap B \cap C) + P(A \cap B \cap C)$

$= P(A)P(B)P(C') + P(A)P(B')P(C) + P(A')P(B)P(C) + P(A)P(B)P(C)$

$= P^2(1-P) + P^2(1-P) + P^2(1-P) + P^3 = 3P^2 - 2P^3$

38. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6 the probability of getting a sum as 3 is

(A) $\frac{1}{18}$
(B) $\frac{5}{18}$
(C) $\frac{1}{5}$
(D) $\frac{2}{5}$

Ans (C)

E = {(1, 1) (1, 2) (1, 3) (1, 4) (2, 1) (2, 2) (2, 3) (3, 1) (3, 2) (4, 1)}

F = {(1, 2) (2, 1)}

$P(F|E) = \frac{2}{10} = \frac{1}{5}$

39. A car manufacturing factory has two plants X and Y. Plant X manufactures 70% of cars and plant Y manufactures 30% of cars. 80% of cars at plant X and 90% of cars at plant Y are rated as standard quality. A car is chosen at random and is found to be of standard quality. The probability that it has come from plant X is

(A) $\frac{56}{73}$ (B) $\frac{56}{84}$ (C) $\frac{56}{83}$ (D) $\frac{56}{79}$

Ans (C)

Let E_1 be the event of plant X manufacturing car

E_2 be the event of plant Y manufacturing car

A be event car is of standard quality

$$P(E_1 | A) = \frac{P(E_1).P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)} = \frac{0.7\times0.8}{0.7\times0.8+0.3\times0.9}$$

$$= \frac{56}{83}$$

40. In a certain town 65% families own cellphones, 15000 families own scooter and 15% families own both. Taking into consideration that the families own at least one of the two, the total number of families in the town is

(A) 20000 (B) 30000 (C) 40000 (D) 50000

Ans (B)

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$x = \frac{65}{100}x + 15,000 - \frac{15}{100}x$$

$x = 0.5x + 15000$

$$\frac{1}{2}x = 15000$$

$x = 30000$

41. A and B are non-singleton sets and $n(A \times B) = 35$. If $B \subset A$ then ${}^{n(A)}C_{n(B)} =$

(A) 28 (B) 35 (C) 42 (D) 21

Ans (D)

$n(A\times B) = 35$

$B \subset A$, $n(B) = 5$, $n(A) = 7$

$$\therefore {}^{n(A)}C_{n(B)} = {}^{7}C_{5} = \frac{7\times6}{2} = 21$$

42. Domain of $f(x) = \frac{x}{1-|x|}$ is

(A) $R - [-1, 1]$ (B) $(-\infty, 1)$ (C) $(-\infty, 1) \cup (0, 1)$ (D) $R - \{-1, 1\}$

Ans (D)

$|x| \neq 1$

$\therefore$ Domain $= R - \{-1, +1\}$

43. The value of cos 1200° + tan 1485° is

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $-\frac{3}{2}$ (D) $-\frac{1}{2}$

Ans (A)

cos 1200° + tan 1485°

= cos [1080° + 120°] + tan [1440° + 45°]

= cos 120° + tan 45°

= cos (180° – 60°) + 1 = – cos 60° + 1 $= -\frac{1}{2} + 1 = \frac{1}{2}$

44. The value of tan 1° tan 2° tan 3° ….. tan 89° is

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) –1

Ans (B)

tan 1°. tan 2° ……. tan 89°

= tan 1°. tan 2° ….. tan 45° ….. cot 2°. cot 1° = 1

45. If $\left(\frac{1+i}{1-i}\right)^x = 1$ then

(A) x = 4n + 1 ; n∈N (B) x = 2n + 1 ; n∈N (C) x = 2n ; n∈N (D) x = 4n ; n∈N

Ans (D)

Given, $\left(\frac{1+i}{1-i}\right)^x = 1$

We know that $\left(\frac{1+i}{1-i}\right) = i$ ⇒ $i^x = 1$ ⇒ $i^{4n} = 1$ ∴ x = 4n, n∈N.

46. The cost and revenue functions of a product are given by c (x) = 20x + 4000 and R(x) = 60x + 2000 respectively where x is the number of items produced and sold. The value of x to earn Profit is

(A) > 50 (B) > 60 (C) > 80 (D) > 40

Ans (A)

R(x) – C(x) > 0

60x + 2000 – 20x – 4000 > 0

40x > 2000

$x > \frac{2000}{40}$

x > 50

47. A student has to answer 10 questions, choosing at least 4 from each of the parts A and B. If there are 6 questions in part A and 7 in part B, then the number of ways can the student choose 10 questions is

(A) 256 (B) 352 (C) 266 (D) 426

Ans (C)

A	B	Number of selections
4	6	${}^6C_4 \times {}^6C_6 = \frac{6\times5}{2}\times7 = 105$
5	5	${}^6C_5 \times {}^7C_5 = 6\times\frac{7\times6}{2} = 126$
6	4	${}^6C_6 \times {}^7C_4 = 1\times\frac{7\times6\times5}{6} = 35$

Total = 105 + 126 + 35 = 266

48. If the middle term of the A.P is 300 then the sum of its first 51 terms is

(A) 15300 (B) 14800 (C) 16500 (D) 14300

Ans (A)

Middle term = $\frac{t_1 + t_n}{2} = 300 \Rightarrow t_1 + t_n = 600$

$\therefore S_{51} = \frac{51}{2}(t_1 + t_n)$

$= \frac{51}{2} \times 600 = 51 \times 300 = 15300$

49. The equation of straight line which passes through the point ($a\cos^3\theta$, $a\sin^3\theta$) and perpendicular to $x\sec\theta + y\,\text{cosec}\,\theta = a$ is

(A) $\frac{x}{a} + \frac{y}{a} = a\cos\theta$ (B) $x\cos\theta - y\sin\theta = a\cos 2\theta$

(C) $x\cos\theta + y\sin\theta = a\cos 2\theta$ (D) $x\cos\theta - y\sin\theta = -a\cos 2\theta$

Ans (B)

$\frac{x}{\sin\theta} - \frac{y}{\cos\theta} = \frac{a\cos^3\theta}{\sin\theta} - \frac{a\sin^3\theta}{\cos\theta}$

$\frac{x\cos\theta - y\sin\theta}{\sin\theta\cos\theta} = \frac{a\cos 2\theta}{\sin\theta\cos\theta}$

$x\cos\theta - y\sin\theta = a\cos 2\theta$

50. The mid points of the sides of a triangle are (1, 5, –1) (0, 4, –2) and (2, 3, 4) then centroid of the triangle

(A) (1, 4, 3) (B) $\left(1, 4, \frac{1}{3}\right)$ (C) (–1, 4, 3) (D) $\left(\frac{1}{3}, 2, 4\right)$

Ans (B)

Centroid = $\left(\frac{1+0+2}{3}, \frac{5+4+3}{3}, \frac{-1-2+4}{3}\right)$

$= \left(1, 4, \frac{1}{3}\right)$

51. Consider the following statements:

Statement 1 : $\lim_{x\to 1}\frac{ax^2 + bx + c}{cx^2 + bx + a}$ is 1 (where $a + b + c \neq 0$)

Statement 2 : $\lim_{x\to -2}\frac{\frac{1}{x} + \frac{1}{2}}{x+2}$ is $\frac{1}{4}$

(A) Only statement 2 is true (B) Only statement 1 is true

(C) Both statements 1 and 2 are true (D) Both statements 1 and 2 are false

Ans (B)

Statement 1

$\lim_{x\to 1}\frac{ax^2 + bx + c}{cx^2 + bx + a}$

$\Rightarrow \frac{a+b+c}{c+b+a} = 1$

Statement 2

$\lim_{x\to -2}\frac{2+x}{2x(x+2)}$

L′ Hospitals Rule $\lim_{x\to -2}\frac{0+1}{4x+4} = \frac{-1}{4}$

52. If a and b are fixed non-zero constants, then the derivative of $\frac{a}{x^4}-\frac{b}{x^2}+\cos x$ is ma + nb − p where

(A) $m = 4x^3$; $n = \frac{-2}{x^3}$; p = sin x
(B) $m = \frac{-4}{x^5}$; $n = \frac{2}{x^3}$; p = sin x
(C) $m = \frac{-4}{x^5}$; $n = \frac{-2}{x^3}$; p = − sin x
(D) $m = 4x^3$; $n = \frac{2}{x^3}$; p = − sin x

Ans (B)

$$\frac{d}{dx}\left(\frac{a}{x^4}-\frac{b}{x^2}+\cos x\right)=ma+nb-p$$

$(-4a)x^{-5} + 2b(x^{-3}) - \sin x = ma + nb - p$

$m = \frac{-4}{x^5}$; $n = \frac{2}{x^3}$; p = sin x

53. The Standard Deviation of the numbers 31, 32, 33 ……. 46, 47 is

(A) $\sqrt{\frac{17}{12}}$
(B) $\sqrt{\frac{47^2-1}{12}}$
(C) $2\sqrt{6}$
(D) $4\sqrt{3}$

Ans (C)

Standard Deviation of the numbers 31, 32, 33 ……. 46, 47 is same as the standard deviation of the numbers 1, 2, 3, ………. 17

Standard Deviation of first 'n' natural numbers = $\sqrt{\frac{n^2-1}{12}}$

Here n = 17

$\therefore$ Standard deviation = $\sqrt{\frac{17^2-1}{12}} = \sqrt{\frac{288}{12}} = 2\sqrt{6}$

54. If P(A) = 0.59, P(B) = 0.30 and P(A ∩ B) = 0.21 then $P(A' \cap B') =$

(A) 0.11
(B) 0.38
(C) 0.32
(D) 0.35

Ans (C)

$P(A' \cap B') = P(A \cup B)'$
$= 1 - P(A \cup B)$
$= 1 - [P(A) + P(B) - P(A \cap B)]$
$= 1 - 0.68$
$= 0.32$

55. f : **R** → **R** defined by $f(x) = \begin{cases} 2x; & x > 3 \\ x^2; & 1 < x \le 3 \\ 3x; & x \le 1 \end{cases}$ then f(−2) + f(3) + f(4) is

(A) 14
(B) 9
(C) 5
(D) 11

Ans (D)

f(−2) + f(3) + f(4)
$\Rightarrow 3(-2) + 3^2 + 2(4)$
$\Rightarrow -6 + 9 + 8 = 11$

56. Let A = {x : x ∈ R; x is not a positive integer} Define f : A → R as $f(x)=\frac{2x}{x-1}$, then f is

(A) injective but not surjective
(B) surjective but not injective
(C) bijective
(D) neither injective nor surjective

Ans (A)

Let A = {x : x ∈ R; x is not a positive integer}

Define f : A → R as $f(x)=\frac{2x}{x-1}$

One-one

$\forall\ x_1, x_2 \in A$

$f(x_1) = f(x_2)$

$\frac{2x_1}{x_1-1}=\frac{2x_2}{x_2-1}$

$x_1x_2 - x_1 = x_1x_2 - x_2$

$x_1 = x_1$

f is one-one

Onto

$y=\frac{2x}{x-1}$

$xy - y = 2x$

$xy - 2x = y$

$x(y - 2) = y$

$x=\frac{y}{y-2}\notin A$

f is not onto

57. The function $f(x)=\sqrt{3}\sin 2x-\cos 2x+4$ is one-one in the interval

(A) $\left[\frac{-\pi}{6},\frac{\pi}{3}\right]$
(B) $\left(\frac{\pi}{6},\frac{-\pi}{3}\right]$
(C) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$
(D) $\left[\frac{-\pi}{6},\frac{-\pi}{3}\right)$

Ans (A)

$f(x)=\sqrt{3}\sin 2x-\cos 2x+4$

$f(x)=2\left\{\frac{\sqrt{3}}{2}\sin x2x-\frac{1}{2}\cos 2x\right\}+4$

$=2\left\{\sin 2x\cos\frac{\pi}{6}-\cos 2x\sin\frac{\pi}{6}\right\}+4$

$=2\left\{\sin\left(2x-\frac{\pi}{6}\right)\right\}+4$

sin x is one-one in $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

$\frac{-\pi}{2}\le 2x-\frac{\pi}{6}\le\frac{\pi}{2}$

$\frac{\pi}{6}-\frac{\pi}{2}\le 2x\le\frac{\pi}{2}+\frac{\pi}{6}$

$\frac{-\pi}{3} \le 2x \le \frac{2\pi}{6}$

$\frac{-\pi}{6} \le x \le \frac{\pi}{3}$

$x \in \left[\frac{-\pi}{6}, \frac{\pi}{3}\right]$

58. Domain of the function $f(x) = \frac{1}{\sqrt{[x]^2 - [x] - 6}}$ where [x] is greatest integer ≤ x is

(A) $(-\infty, -2) \cup [4, \infty]$ (B) $(-\infty, -2) \cup [3, \infty]$

(C) $[-\infty, -2] \cup [4, \infty]$ (D) $[-\infty, -2] \cup [3, \infty)$

Ans Grace

$f(x) = \frac{1}{\sqrt{[x]^2 - [x] - 6}}$

$[x]^2 - [x] - 6 > 0$

$([x] - 3)([x] + 2) > 0$

$[x] < -2$ or $[x] > 3$

$(-\infty, -2) \cup [4, \infty)$

59. $\cos\left[\cot^{-1}\left(-\sqrt{3}\right) + \frac{\pi}{6}\right] =$

(A) 0 (B) 1 (C) $\frac{1}{\sqrt{2}}$ (D) − 1

Ans (D)

$\cos\left[\cot^{-1}\left(-\sqrt{3}\right) + \frac{\pi}{6}\right]$

$= \cos\left[\pi - \frac{\pi}{6} + \frac{\pi}{6}\right] = \cos \pi = -1$

60. $\tan^{-1}\left[\frac{1}{\sqrt{3}} \sin\frac{5\pi}{2}\right] \sin^{-1}\left[\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right] =$

(A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) π

Ans Grace

$\tan^{-1}\left[\frac{1}{\sqrt{3}} \sin\frac{5\pi}{2}\right] \sin^{-1}\left[\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right]$

$= \tan^{-1}\left[\frac{1}{\sqrt{3}} \sin\left(2\pi + \frac{\pi}{2}\right)\right] \sin^{-1}\left[\cos\left(\frac{\pi}{3}\right)\right]$

$= \tan^{-1}\left[\frac{1}{\sqrt{3}}(1)\right] \sin^{-1}\left[\frac{1}{2}\right]$

$= \frac{\pi}{6} \cdot \frac{\pi}{6} = \frac{\pi^2}{36}$

* * *